Math 10B - Calculus of Several Variables II
Quiz 5 Make Up
Due June 3, 2011

Name: ______________________________

Instructions: This will allow you to make up points for quiz 5, raising your score to at most 9 out of 10. Do as many parts as you deem necessary. Each part is worth 2 points. This will be due in discussion on Friday at the beginning of class. Attach this as a coversheet to what you turn in and clearly indicate which problems you attempted by putting a 0 in the score boxes which you did not do the problem for.

1	2	3	4	5	Total
/2	/2	/2	/2	/2	/10

Problem 1. *Verify that Stokes' theorem implies Green's theorem (including the technical assumptions). (Hint: In Stokes' theorem, take* $\mathbf{F}(x,y,z) = (M(x,y), N(x,y), 0)$*. That is, assume that* $\mathbf{F}$ *is independend of* z*, and that its* $\mathbf{k}$*-component is zero.)*

Problem 2. *Give a proof of Stokes' theorem for smooth, parametrized surfaces* $S = \mathbf{X}(D)$*, where* $\mathbf{X} : D \subset \mathbb{R}^2 \to \mathbb{R}^3$*. To make the proof easier, assume that* $\mathbf{X}$ *is of class* $\mathcal{C}^2$ *and that is it one-one on* D *(in which case* $\partial S = \mathbf{X}(\partial D)$*).*

Problem 3. *Suppose that a surface is given in cylindrical coordinates by the equation* $z = f(r,\theta)$*, where* (r,θ) *varies through a region* D *in the* $r\theta$*-plane where* r *is nonnegative. Show that the surface area of the surface is given by*

$$\iint_D \sqrt{1 + \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2}\, r\,dr\,d\theta.$$

Problem 4. *Show that the surface parametrized by:*

$$\begin{cases} x &= \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\cos s \\ y &= \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\sin s \\ z &= \sin\frac{s}{2}\sin t + \cos\frac{s}{2}\sin 2t \end{cases}$$

is not orientable.

Problem 5. *Let* $f, g : R \subset \mathbb{R}^3 \to \mathbb{R}$ *be of class* $\mathcal{C}^2$*, and let* D *be a solid region in space contained in* R*, bounded by a piecewise smooth surface* $S = \partial D$*. Let* S *be oriented with outward normals (pointing away from* D*). Prove that:*

$$\iiint_D \nabla f \cdot \nabla g \, dV + \iiint_D f \nabla^2 g \, dV = \oiint_S f \nabla g \cdot d\mathbf{S}.$$

(Hint: Use Gauss' theorem.)